18.100A PROBLEM SET 6

due May 3rd 9:30 am

You can collaborate with other students when working on problems. However, you should write the solutions using your own words and thought.

Problem 1. *$f(x)$ and $g(x)$ are integrable on $[a,b]$, and $g(x)$ is bounded below by a positive constant $c>0$. Prove that $\frac{f(x)}{g(x)}$ is integrable.*

Problem 2. *Find some functions $f(x)$ and $g(x)$ satisfying the hypothesis in the* **Problem 1** *but violating the following identity*

$$\int_a^b \frac{f(x)}{g(x)}dx = \frac{\int_a^b f(x)dx}{\int_a^b g(x)dx}\,.$$

Problem 3. *Exercise 19.3.1. Page 263.*

Problem 4. *Exercise 19.4.4. Page 264.*

Problem 5. *Exercise 21.1.3.(a) Page 299.*

Problem 6 (20 points)**.** *Exercise 21.2.1.(a)(e)(f)(h) Page 300.*

Problem 7. *Exercise 21.2.5. Page 300.*

Problem 8. *Exercise 21.2.6. Page 300.*

Problem 9. *Assume f is continuous on $(0,1)$. Prove that if $\int_{0^+}^{1^-} |f(x)|^p dx$ converges for some $p>1$, then $\int_{0^+}^{1^-} f(x)dx$ converges.*

(Hint: Use Young's inequality for products given in a supplementary note.)

Problem 10. *Exercise 21.4.3. Page 301.*

Problem 11. *Problem 21-3 Page 301.*